

# In a nutshell: The secant method

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Given a continuous real-valued function  $f$  of a real variable with two initial approximations of a root  $x_{-1}$  and  $x_0$  where  $|f(x_{-1})| > |f(x_0)| > 0$ , swapping them if  $|f(x_0)| > |f(x_{-1})| > 0$ . If they are equal, this algorithm will not work, and if one is zero, we have already found a root. This algorithm uses iteration, linear interpolation and solving a trivial linear equation to approximate a root.

Parameters:

$\epsilon_{\text{step}}$	The maximum error in the value of the root cannot exceed this value.
$\epsilon_{\text{abs}}$	The value of the function at the approximation of the root cannot exceed this value.
$N$	The maximum number of iterations.

1. Let  $k \leftarrow 0$ .
2. If  $k > N$ , we have iterated  $N$  times, so stop and return signalling a failure to converge.
3. If  $|f(x_{k-1})| < |f(x_k)|$ , swap these two values.
4. The next approximation to the root will be the root of the linear polynomial that interpolates the points  $(x_{k-1}, f(x_{k-1}))$  and  $(x_k, f(x_k))$ .

$$\text{Let } x_{k+1} \leftarrow x_k - f(x_k) \frac{x_{k-1} - x_k}{f(x_{k-1}) - f(x_k)}.$$

- a. If  $x_{k+1}$  is not a finite floating-point number, so return signalling a failure to converge.
  - b. If  $|x_{k+1} - x_k| < \epsilon_{\text{step}}$  and  $|f(x_{k+1})| < \epsilon_{\text{abs}}$ , return  $x_{k+1}$ .
5. Increment  $k$  and return to Step 2.

## Convergence

If  $h$  is the error, it can be show that the error decreases according to  $O(h^\phi)$  where  $\phi \approx 1.6180$  is the golden ratio (the positive root of  $x^2 - x - 1 = 0$ ). This technique is not guaranteed to converge if there is a root, for the denominator could be arbitrarily small, causing the next approximation to be arbitrarily far from the previous approximation.